Can Decentralization Improve Social Welfare?

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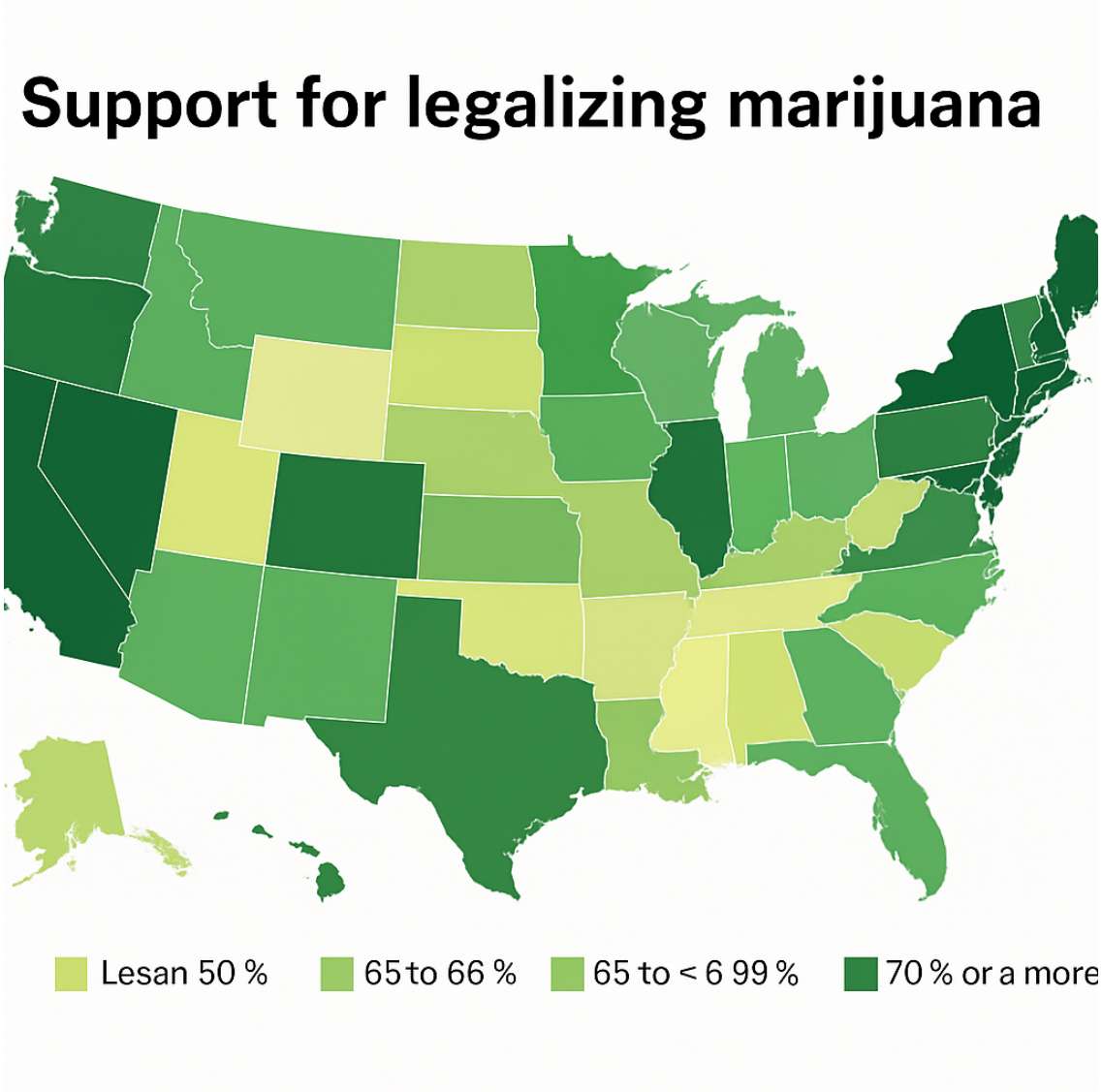
July 15, 2025

Outline

A case study based on poll data



Why Study Policy Coordination on Graphs?

- States/agents interact with neighbors in a network.
- However, former studies don't care **border cost** and are not quantified.
- **Trade-off**: Local preference vs border cost.
- We also study **dynamic evolution** of network.

Model Definition

Graph Structure

$\mathcal{G} = (\mathcal{P}, \mathcal{L}, \mathcal{F}, u, c)$

- Two policies: $pol_i \in \{0, 1\}$, for $i \in \{1, 2, \ldots, n\}$.
- Preferences $f_i = (f_{i,1}, f_{i,2})$.
- Utility:

$$u_i = \begin{cases} f_{i,1} & \text{if } pol_i = 1, \\ f_{i,2} & \text{if } pol_i = 0. \end{cases}$$

- Cost:

$$c_{ij} = \begin{cases} 0 & pol_i = pol_j, \\ f_{i,2} + f_{j,1} & pol_i = 1, pol_j = 0, \\ f_{i,1} + f_{j,2} & pol_i = 0, pol_j = 1. \end{cases}$$

Social Welfare and Optimization

- Total socail welfare:

$$\tau = \sum_i \left(u_i - \frac{1}{2} \sum_j{}' c_{ij} \right)$$

- Goal: Maximize τ over all policy profiles, i.e.

$$pol = \arg \max_{pol \in \mathrm{POL}} \tau.$$

Definition

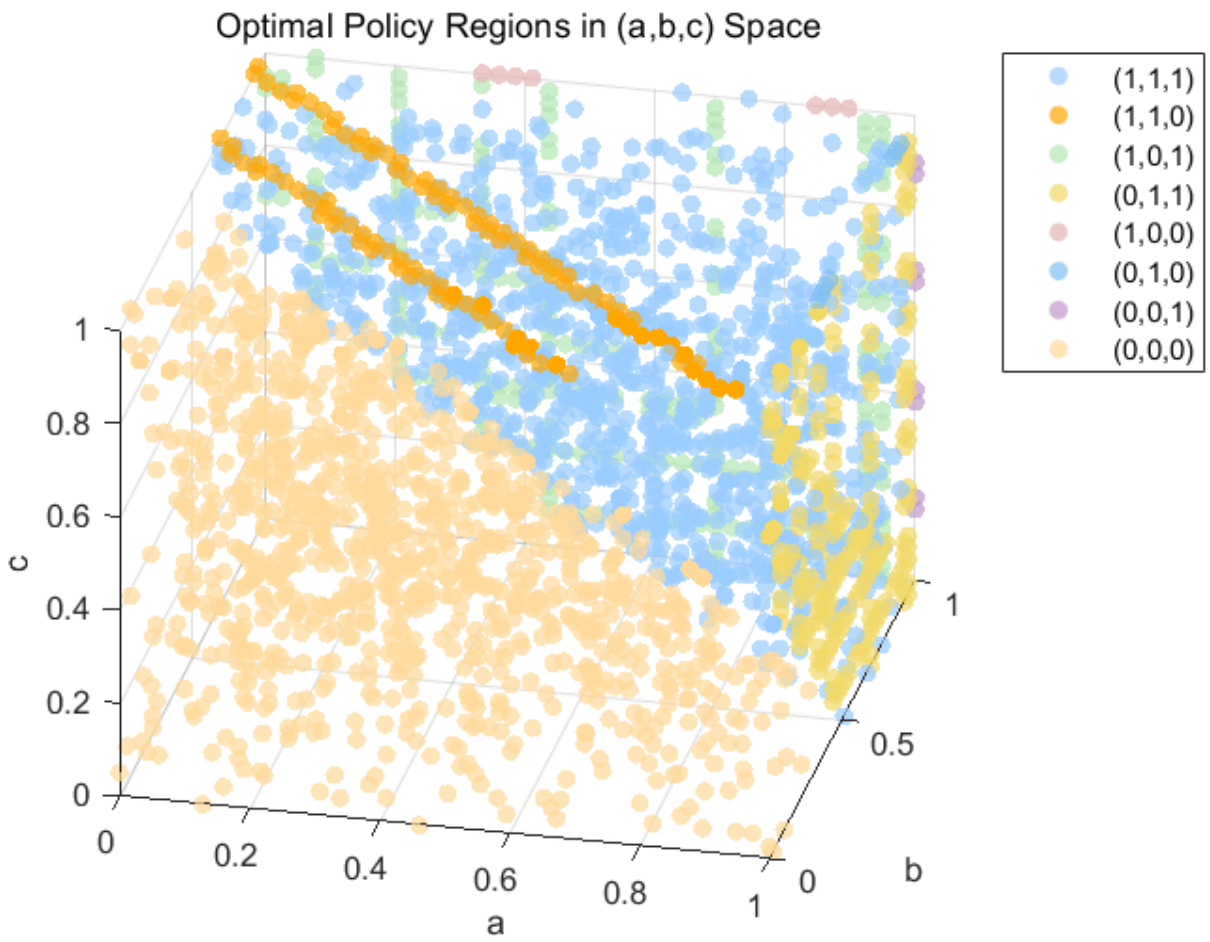
$$\mu(\mathcal{G}) = \max_{pol \notin \{\mathbf{0},\mathbf{1}\}} \tau - \max_{pol \in \{\mathbf{0},\mathbf{1}\}} \tau$$

- Measures welfare gain from decentralized policies.

Three-State Case

- States 1, 2, 3 with preferences $(a, 1-a), (b, 1-b), (c, 1-c)$.
- Complete graph.
- Compute τ for each of 8 policy profiles.

Preference Region Visualization



- Clear boundary: $a + b + c = 1.5$.
- The powerful cost terms drives coordination behavior.

Model Extension

- Introduce $b_{ij} \in [0, 1]$ to control c_{ij}.
- When $b_{ij} = 0$: fully decentralized (no cross-border cost).
- When $b_{ij} = 1$: fully centralized model.

$$\tau = \sum_i \left(u_i - \frac{1}{2} \sum_j{}' b_{ij} c_{ij} \right)$$

Dynamic Model

$$\begin{cases} \dot{f}_{i,1}(t) = \eta \cdot (1 - f_{i,1}(t)), \\ \dot{f}_{i,2}(t) = -\eta \cdot f_{i,2}(t), \end{cases} \quad \text{if } pol_i(t) = 1; \tag{1}$$

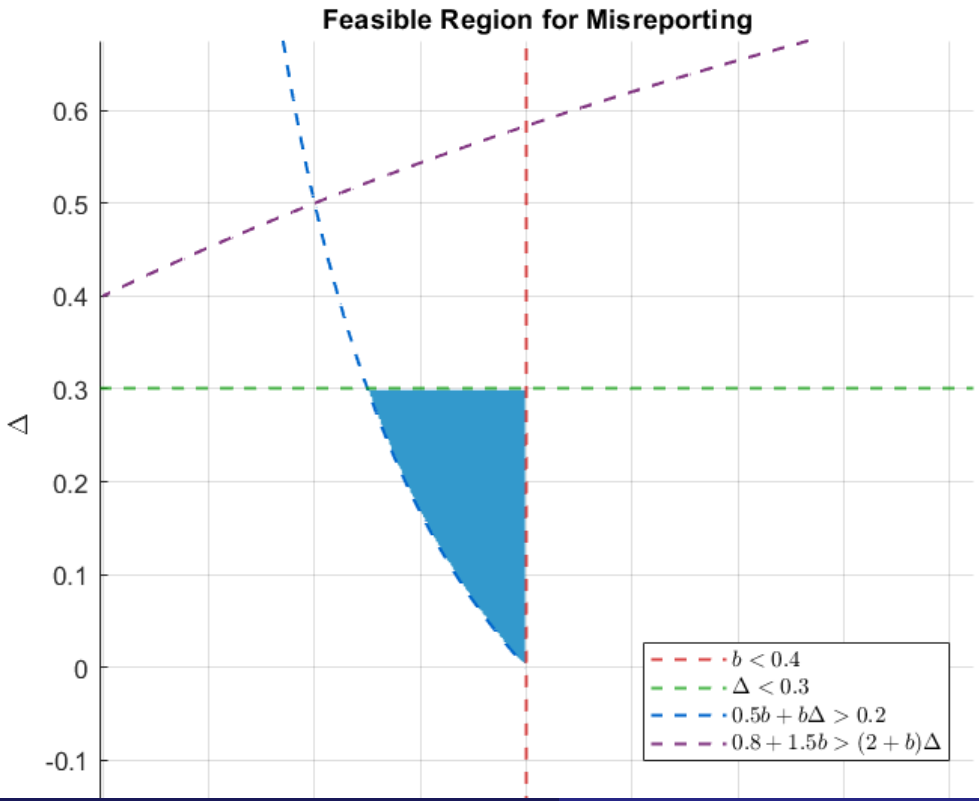
$$\begin{cases} \dot{f}_{i,1}(t) = -\eta \cdot f_{i,1}(t), \\ \dot{f}_{i,2}(t) = \eta \cdot (1 - f_{i,2}(t)), \end{cases} \quad \text{if } pol_i(t) = 2. \tag{2}$$

- Policy will influence regional preferences.

Competition and Strategy

Example (Misreporting in Two-state Model)

Let state 1 have a preference of $f_{1,1} = 0.9$ for policy 1, and state 2 have a preference of $f_{2,1} = 0.4$. Let b_{12} denote the border influence coefficient. Here we consider party A misreporting some of its supporters in state 1, whose number is Δ .



Summary and Outlook

- Our work combines total welfare, dynamics and competition.
- Future: defining Nash Equilibrium, complexity analysis and real data modification.